

4.3

Proving Triangles are Congruent: SSS and SAS

What you should learn

GOAL 1 Prove that triangles are congruent using the SSS and SAS Congruence Postulates.

GOAL 2 Use congruence postulates in **real-life** problems, such as bracing a structure in **Example 5**.

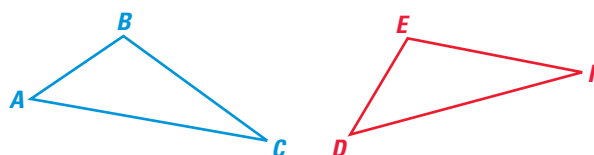
Why you should learn it

▼ Congruence postulates help you see why triangles make things stable, such as the seaplane's wing below and the objects in Exs. 30 and 31.



GOAL 1 SSS AND SAS CONGRUENCE POSTULATES

How much do you need to know about two triangles to prove that they are congruent? In Lesson 4.2, you learned that if all six pairs of corresponding parts (sides and angles) are congruent, then the triangles are congruent.



If	Sides are congruent	and	Angles are congruent	then	Triangles are congruent
	1. $\overline{AB} \cong \overline{DE}$		4. $\angle A \cong \angle D$		$\triangle ABC \cong \triangle DEF$
	2. $\overline{BC} \cong \overline{EF}$		5. $\angle B \cong \angle E$		
	3. $\overline{AC} \cong \overline{DF}$		6. $\angle C \cong \angle F$		

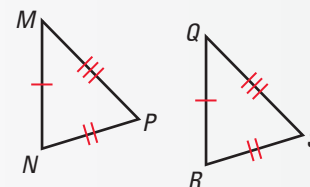
In this lesson and the next, you will learn that you do not need all six of the pieces of information above to prove that the triangles are congruent. For example, if all three pairs of corresponding sides are congruent, then the *SSS Congruence Postulate* guarantees that the triangles are congruent.

POSTULATE

POSTULATE 19 Side-Side-Side (SSS) Congruence Postulate

If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

If Side $\overline{MN} \cong \overline{QR}$,
 Side $\overline{NP} \cong \overline{RS}$, and
 Side $\overline{PM} \cong \overline{SQ}$,
 then $\triangle MNP \cong \triangle QRS$.

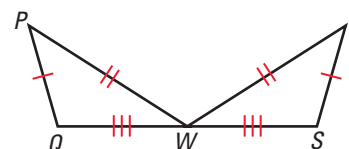


EXAMPLE 1 Using the SSS Congruence Postulate

Prove that $\triangle PQW \cong \triangle TSW$.

Paragraph Proof The marks on the diagram show that $\overline{PQ} \cong \overline{TS}$, $\overline{PW} \cong \overline{TW}$, and $\overline{QW} \cong \overline{SW}$.

► So, by the SSS Congruence Postulate, you know that $\triangle PQW \cong \triangle TSW$.

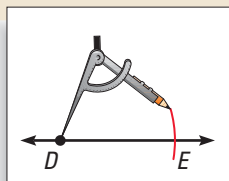
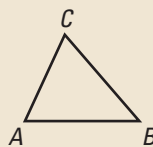


ACTIVITY

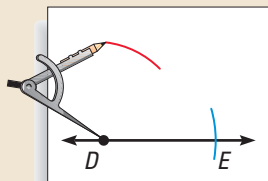
Construction

Copying a Triangle

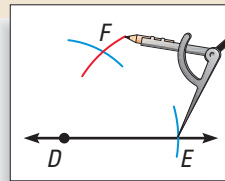
Follow the steps below to construct a triangle that is congruent to a given $\triangle ABC$.



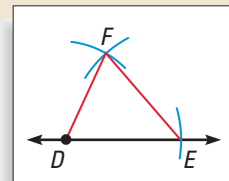
- 1** Construct \overline{DE} so that it is congruent to \overline{AB} . (See page 104 for the construction.)



- 2** Open your compass to the length AC . Use this length to draw an arc with the compass point at D .



- 3** Draw an arc with radius BC and center E that intersects the arc from **Step 2**. Label the intersection point F .

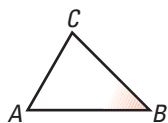


- 4** Draw $\triangle DEF$. By the SSS Congruence Postulate, $\triangle ABC \cong \triangle DEF$.

The SSS Congruence Postulate is a shortcut for proving two triangles are congruent without using all six pairs of corresponding parts. The postulate below is a shortcut that uses two sides and the angle that is *included* between the sides.

STUDENT HELP

Study Tip
In the triangle, $\angle B$ is the **included angle** between sides \overline{AB} and \overline{BC} .

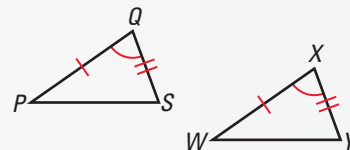


POSTULATE

POSTULATE 20 Side-Angle-Side (SAS) Congruence Postulate

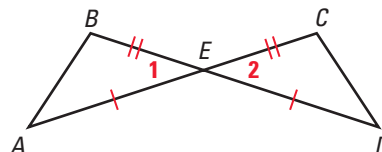
If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

If **Side** $\overline{PQ} \cong \overline{WX}$,
Angle $\angle Q \cong \angle X$, and
Side $\overline{QS} \cong \overline{XY}$,
then $\triangle PQS \cong \triangle WXY$.



EXAMPLE 2 Using the SAS Congruence Postulate

Prove that $\triangle AEB \cong \triangle DEC$.



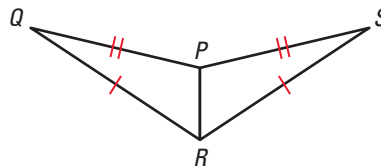
Statements	Reasons
1. $\overline{AE} \cong \overline{DE}$, $\overline{BE} \cong \overline{CE}$	1. Given
2. $\angle 1 \cong \angle 2$	2. Vertical Angles Theorem
3. $\triangle AEB \cong \triangle DEC$	3. SAS Congruence Postulate

GOAL 2 MODELING A REAL-LIFE SITUATION



EXAMPLE 3 Choosing Which Congruence Postulate to Use

Decide whether enough information is given in the diagram to prove that $\triangle PQR \cong \triangle PSR$. If there is enough information, state the congruence postulate you would use.



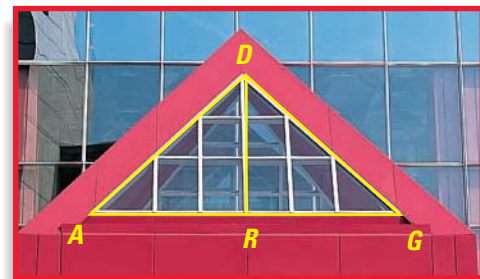
SOLUTION

Paragraph Proof The marks on the diagram show that $\overline{PQ} \cong \overline{PS}$ and $\overline{QR} \cong \overline{SR}$. By the Reflexive Property of Congruence, $\overline{RP} \cong \overline{RP}$. Because the sides of $\triangle PQR$ are congruent to the corresponding sides of $\triangle PSR$, you can use the SSS Congruence Postulate to prove that the triangles are congruent.

EXAMPLE 4 Proving Triangles Congruent



ARCHITECTURE You are designing the window shown in the photo. You want to make $\triangle DRA$ congruent to $\triangle DRG$. You design the window so that $\overline{DR} \perp \overline{AG}$ and $\overline{RA} \cong \overline{RG}$. Can you conclude that $\triangle DRA \cong \triangle DRG$?



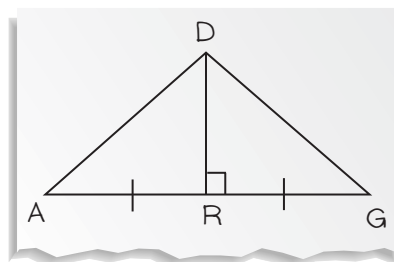
SOLUTION

To begin, copy the diagram and label it using the given information. Then write the given information and the statement you need to prove.



GIVEN $\overline{DR} \perp \overline{AG}$,
 $\overline{RA} \cong \overline{RG}$

PROVE $\triangle DRA \cong \triangle DRG$



Statements	Reasons
1. $\overline{DR} \perp \overline{AG}$	1. Given
2. $\angle DRA$ and $\angle DRG$ are right angles.	2. If 2 lines are \perp , then they form 4 rt. \angle s.
3. $\angle DRA \cong \angle DRG$	3. Right Angle Congruence Theorem
4. $\overline{RA} \cong \overline{RG}$	4. Given
5. $\overline{DR} \cong \overline{DR}$	5. Reflexive Property of Congruence
6. $\triangle DRA \cong \triangle DRG$	6. SAS Congruence Postulate

EXAMPLE 5 *Triangular Frameworks are Rigid*

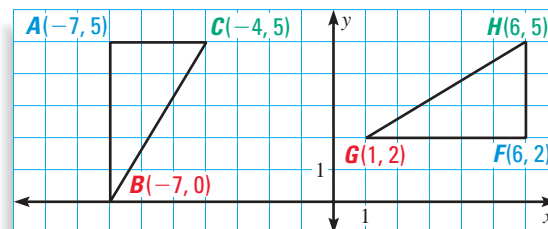
STRUCTURAL SUPPORT To prevent a doorway from collapsing after an earthquake, you can reinforce it. Explain why the doorway with the diagonal brace is more stable, while the one without the brace can collapse.

**SOLUTION**

In the doorway with the diagonal brace, the wood forms triangles whose sides have fixed lengths. The SSS Congruence Postulate guarantees that these triangles are rigid, because a triangle with given side lengths has only one possible size and shape. The doorway without the brace is unstable because there are many possible shapes for a four-sided figure with the given side lengths.

**EXAMPLE 6** *Congruent Triangles in a Coordinate Plane*

Use the SSS Congruence Postulate to show that $\triangle ABC \cong \triangle FGH$.

**SOLUTION**

Because $AC = 3$ and $FH = 3$, $\overline{AC} \cong \overline{FH}$. Because $AB = 5$ and $FG = 5$, $\overline{AB} \cong \overline{FG}$. Use the Distance Formula to find the lengths BC and GH .

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\begin{aligned} BC &= \sqrt{(-4 - (-7))^2 + (5 - 0)^2} \\ &= \sqrt{3^2 + 5^2} \\ &= \sqrt{34} \end{aligned}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\begin{aligned} GH &= \sqrt{(6 - 1)^2 + (5 - 2)^2} \\ &= \sqrt{5^2 + 3^2} \\ &= \sqrt{34} \end{aligned}$$

STUDENT HELP**Look Back**

For help with the Distance Formula, see page 19.

- Because $BC = \sqrt{34}$ and $GH = \sqrt{34}$, $\overline{BC} \cong \overline{GH}$. All three pairs of corresponding sides are congruent, so $\triangle ABC \cong \triangle FGH$ by the SSS Congruence Postulate.

GUIDED PRACTICE

Vocabulary Check ✓

1. Sketch a triangle and label its vertices. Name two sides and the included angle between the sides.

Concept Check ✓

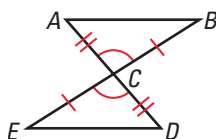
2. **ERROR ANALYSIS** Henry believes he can use the information given in the diagram and the SAS Congruence Postulate to prove the two triangles are congruent. Explain Henry's mistake.



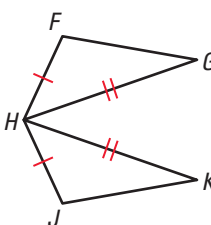
Skill Check ✓

- LOGICAL REASONING** Decide whether enough information is given to prove that the triangles are congruent. If there is enough information, tell which congruence postulate you would use.

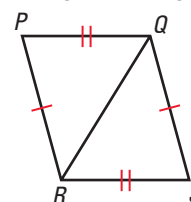
3. $\triangle ABC$, $\triangle DEC$



4. $\triangle FGH$, $\triangle JKH$



5. $\triangle PQR$, $\triangle SRQ$



PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice
to help you master
skills is on p. 809.

- NAMING SIDES AND INCLUDED ANGLES** Use the diagram. Name the included angle between the pair of sides given.

6. \overline{JK} and \overline{KL}

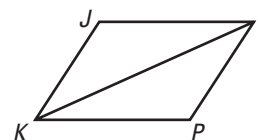
7. \overline{PK} and \overline{LK}

8. \overline{LP} and \overline{LK}

9. \overline{JL} and \overline{JK}

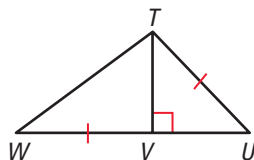
10. \overline{KL} and \overline{JL}

11. \overline{KP} and \overline{PL}

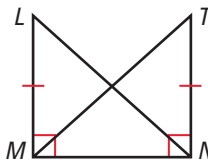


- LOGICAL REASONING** Decide whether enough information is given to prove that the triangles are congruent. If there is enough information, state the congruence postulate you would use.

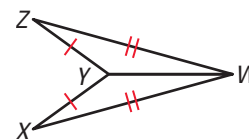
12. $\triangle UVT$, $\triangle WVT$



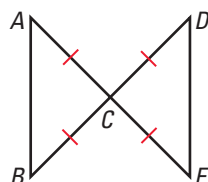
13. $\triangle LMN$, $\triangle TNM$



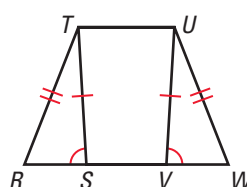
14. $\triangle YZW$, $\triangle YXW$



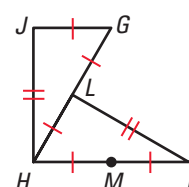
15. $\triangle ACB$, $\triangle ECD$



16. $\triangle RST$, $\triangle WVU$



17. $\triangle GJH$, $\triangle HLK$



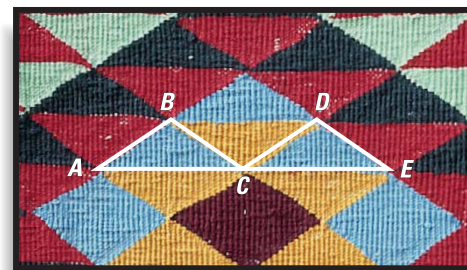
STUDENT HELP

HOMEWORK HELP

- Example 1:** Exs. 18, 20–28
Example 2: Exs. 19–28
Example 3: Exs. 12–17
Example 4: Exs. 20–28
Example 5: Exs. 30, 31
Example 6: Exs. 33–35

DEVELOPING PROOF In Exercises 18 and 19, use the photo of the Navajo rug. Assume that $\overline{BC} \cong \overline{DE}$ and $\overline{AC} \cong \overline{CE}$.

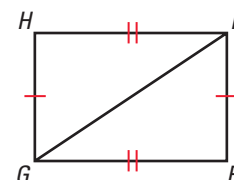
18. What other piece of information is needed to prove that $\triangle ABC \cong \triangle CDE$ using the SSS Congruence Postulate?
19. What other piece of information is needed to prove that $\triangle ABC \cong \triangle CDE$ using the SAS Congruence Postulate?



20. **DEVELOPING PROOF** Complete the proof by supplying the reasons.

GIVEN $\overline{EF} \cong \overline{GH}$,
 $\overline{FG} \cong \overline{HE}$

PROVE $\triangle EFG \cong \triangle GHE$

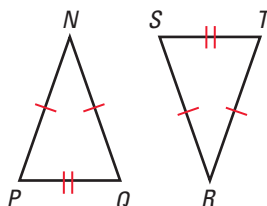


Statements	Reasons
1. $\overline{EF} \cong \overline{GH}$	1. ?
2. $\overline{FG} \cong \overline{HE}$	2. ?
3. $\overline{GE} \cong \overline{GE}$	3. ?
4. $\triangle EFG \cong \triangle GHE$	4. ?

TWO-COLUMN PROOF Write a two-column proof.

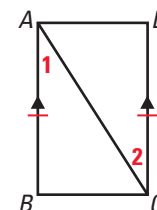
21. **GIVEN** $\overline{NP} \cong \overline{QN} \cong \overline{RS} \cong \overline{TR}$,
 $\overline{PQ} \cong \overline{ST}$

PROVE $\triangle NPQ \cong \triangle RST$



22. **GIVEN** $\overline{AB} \cong \overline{CD}$, $\overline{AB} \parallel \overline{CD}$

PROVE $\triangle ABC \cong \triangle CDA$



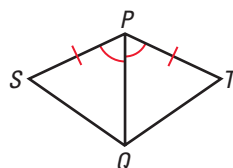
STUDENT HELP

INTERNET
HOMEWORK HELP
Visit our Web site
www.mcdougallittell.com
for help with paragraph
proofs.

PARAGRAPH PROOF Write a paragraph proof.

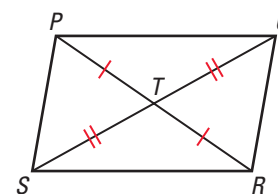
23. **GIVEN** \overline{PQ} bisects $\angle SPT$,
 $\overline{SP} \cong \overline{TP}$

PROVE $\triangle SPQ \cong \triangle TPQ$



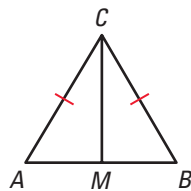
24. **GIVEN** $\overline{PT} \cong \overline{RT}$, $\overline{QT} \cong \overline{ST}$

PROVE $\triangle PQT \cong \triangle RST$

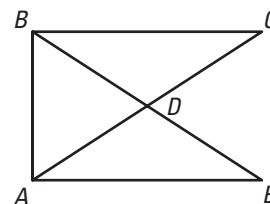


PROOF Write a two-column proof or a paragraph proof.

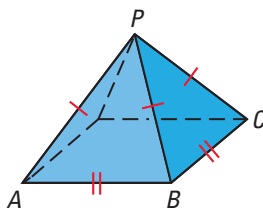
25. **GIVEN** $\overline{AC} \cong \overline{BC}$,
 M is the midpoint of \overline{AB} .
PROVE $\triangle ACM \cong \triangle BCM$



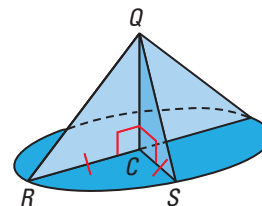
26. **GIVEN** $\overline{BC} \cong \overline{AE}$, $\overline{BD} \cong \overline{AD}$,
 $\overline{DE} \cong \overline{DC}$
PROVE $\triangle ABC \cong \triangle BAE$



27. **GIVEN** $\overline{PA} \cong \overline{PB} \cong \overline{PC}$,
 $\overline{AB} \cong \overline{BC}$
PROVE $\triangle PAB \cong \triangle PBC$

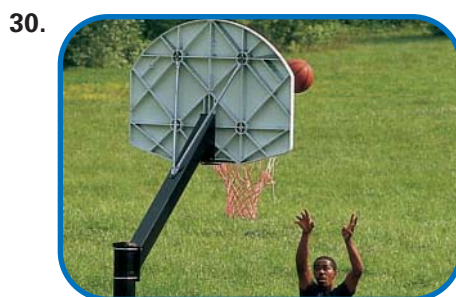


28. **GIVEN** $\overline{CR} \cong \overline{CS}$, $\overline{QC} \perp \overline{CR}$,
 $\overline{QC} \perp \overline{CS}$
PROVE $\triangle QCR \cong \triangle QCS$



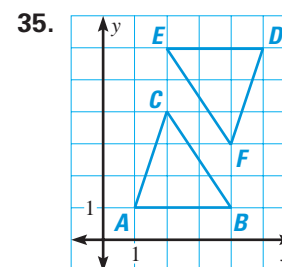
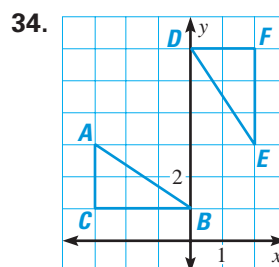
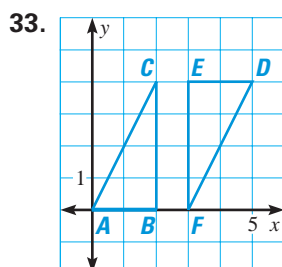
29. **TECHNOLOGY** Use geometry software to draw a triangle. Draw a line and reflect the triangle across the line. Measure the sides and the angles of the new triangle and tell whether it is congruent to the original one.

Writing Explain how triangles are used in the object shown to make it more stable.



32. **CONSTRUCTION** Draw an isosceles triangle with vertices A , B , and C . Use a compass and straightedge to construct $\triangle DEF$ so that $\triangle DEF \cong \triangle ABC$.

USING ALGEBRA Use the Distance Formula and the SSS Congruence Postulate to show that $\triangle ABC \cong \triangle DEF$.



STUDENT HELP

SOFTWARE HELP
 Visit our Web site
www.mcdougallittell.com
 to see instructions for
 several software
 applications.

Test Preparation

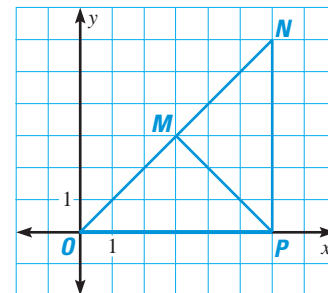
36. **MULTIPLE CHOICE** In $\triangle RST$ and $\triangle ABC$, $\overline{RS} \cong \overline{AB}$, $\overline{ST} \cong \overline{BC}$, and $\overline{TR} \cong \overline{CA}$. Which angle is congruent to $\angle T$?
- (A) $\angle R$ (B) $\angle A$ (C) $\angle C$ (D) cannot be determined
37. **MULTIPLE CHOICE** In equilateral $\triangle DEF$, a segment is drawn from point F to G , the midpoint of \overline{DE} . Which of the statements below is *not* true?
- (A) $\overline{DF} \cong \overline{EF}$ (B) $\overline{DG} \cong \overline{DF}$ (C) $\overline{DG} \cong \overline{EG}$ (D) $\triangle DFG \cong \triangle EFG$

★ Challenge

EXTRA CHALLENGE

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38. **CHOOSING A METHOD** Describe how to show that $\triangle PMO \cong \triangle PMN$ using the SSS Congruence Postulate. Then find a way to show that the triangles are congruent using the SAS Congruence Postulate. You may not use a protractor to measure any angles. Compare the two methods. Which do you prefer? Why?



MIXED REVIEW

SCIENCE CONNECTION Find an important angle in the photo. Copy the angle, extend its sides, and use a protractor to measure it to the nearest degree. (Review 1.4)

39.

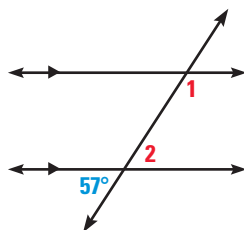


40.

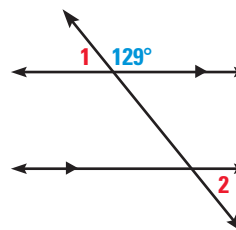


USING PARALLEL LINES Find $m\angle 1$ and $m\angle 2$. Explain your reasoning. (Review 3.3 for 4.4)

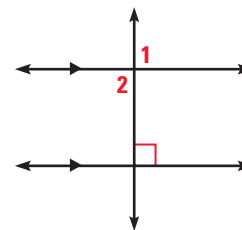
41.



42.

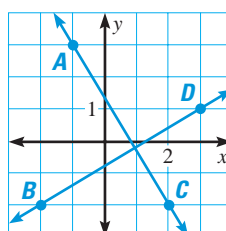


43.

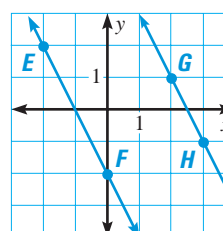


LINE RELATIONSHIPS Find the slope of each line. Identify any parallel or perpendicular lines. (Review 3.7)

44.



45.



46.

